Fall 2013 **Math 601** Name: ____________________

Quiz 4

Question 1. (10 pts)

Determine whether the following statements are true or false. If false, explain why.

(a) If a vector space V has dimension n, then any $n+1$ vectors in V must span V.

> **Solution:** False. For example, choose the same vector $n+1$ times. Then they certainly do not span V, in the case when $n>1$.

(b) Let V be a vector space of dimension n. Suppose $\{u_1, \cdots, u_m\}$ is linearly independent. Then $m \le n$.

> **Solution:** True.

(c) Let V be a vector space. If $\{v_1, \cdots, v_k\}$ spans V, then there is a subset of $\{v_1, \cdots, v_k\}$ that forms a basis of V.

> **Solution:** True.

(d) The polynomials $u_1 = 2, u_2 = (t-2), u_3 = (t-2)^2$ form a basis of $\mathbf{P}_2(t)$.

> **Solution:** True.

(e) Given an 3×4 matrix A, suppose $\text{rank}(A) = 2$. Then the rows of A are linearly dependent.

> **Solution:** True.

Question 2. (10 pts)
Given

$$A = \begin{bmatrix} 1 & 2 & 3 & 2 & 1 \\ 3 & 6 & 9 & 6 & 3 \\ 1 & 2 & 4 & 1 & 2 \\ 2 & 4 & 9 & 1 & 2 \end{bmatrix}$$

(a) (5 pts) Find a basis for the row space of A.

Solution: Use Gaussian elimination to find an echelon matrix of A

$$\begin{bmatrix} 1 & 2 & 3 & 2 & 1 \\ 0 & 0 & 3 & -3 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

So $u_1 = (1, 2, 3, 2, 1)$, $u_2 = (0, 0, 3, -3, 0)$ and $u_3 = (0, 0, 0, 0, 1)$ form a basis of the row space of A.

(b) (5 pts) Find a basis for the column space of A.

Solution: Use the same echelon matrix above. Then we see that

$$v_1 = \begin{bmatrix} 1 \\ 3 \\ 1 \\ 2 \end{bmatrix}, v_2 = \begin{bmatrix} 3 \\ 9 \\ 4 \\ 9 \end{bmatrix}, v_3 = \begin{bmatrix} 1 \\ 3 \\ 2 \\ 2 \end{bmatrix}$$

form a basis of the column space of A.